***On Semi-Supervised Fuzzy c-Means Clustering***

Yasunori et al. (2009) have introduced novel fuzzy C-Means clustering algorithms that incorporate aspects of supervised classification by integrating supervised membership degrees into the fuzzy C-Means framework. In semi-supervised Fuzzy C-Means clustering, certain degrees of membership for a subset of data points are determined in advance. These supervised membership degrees, referred to as , represent the influence of supervision and focus on the membership degrees assigned to specific clusters rather than the data in the sample space. Assigning is equivalent to labeling a data point as a supervisor for cluster . The authors proposed two semi-supervised fuzzy C-Means clustering algorithms using . One is based on the standard fuzzy C-means and the other is based on the entropy-regularized fuzzy C-means. One notable benefit of this algorithm is its practical classification results, which are realized through a methodology that is less complex than traditional semi-supervised clustering methods.

1. *Semi-Supervised Standard Fuzzy c-Means Clustering*

This section aims to develop a semi-supervised version of the standard fuzzy c-means clustering algorithm by incorporating the supervised membership degree into the traditional fuzzy c-means framework. To achieve this, the following objective function is minimized, which is formulated by introducing into the objective function of the standard fuzzy c-means algorithm:

|  |  |
| --- | --- |
|  | (1) |

The constraints are given as follows:

|  |  |
| --- | --- |
|  | (2) |
|  | (3) |

First, the optimal solution is calculated to minimizes

|  |  |
| --- | --- |
|  | (4) |

Next, the optimal solution for that minimizes is derived. Two cases need to be considered: and . This is because, when , **F** becomes piecewise linear and non-differentiable at .

We consider the case For this case, the objective function is defined as follows.

|  |  |
| --- | --- |
|  | (5) |

where the constraints for and the conditions for are given by (2) and (3), respectively. the optimal solution is calculated as follows:

|  |  |
| --- | --- |
|  | (6) |

It is important to note that:

|  |
| --- |
|  |

The solution is derived from (5), and it is known that .

*B. Semi-Supervised Entropy Regularized Fuzzy c-Means Clustering*

In this section, the aim is to develop a semi-supervised entropy-regularized fuzzy C-Means clustering algorithm by integrating the supervised membership degree into the entropy-regularized fuzzy C-Means framework. To achieve this, the following objective function is minimized, which is formulated by introducing into the objective function of the entropy-regularized fuzzy C-Means cluster considering the constraints presented in (3).

|  |  |
| --- | --- |
|  | (7) |

First, the optimal solution is obtained to minimizes

|  |  |
| --- | --- |
|  | (8) |

Next, the optimal solution for is calculated to minimizes . In contrast to (7), the subsequent objective function is formulated which eliminates the use of absolute values:

|  |  |
| --- | --- |
|  | (9) |

Thus, the following optimal solution is obtained:

|  |  |
| --- | --- |
|  | (10) |